The Pythagorean Theorem

**MAIN IDEA**
Find length using the Pythagorean Theorem.

**New Vocabulary**
- leg
- hypotenuse
- Pythagorean Theorem

**Math Online**
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- Extra Examples
- Personal Tutor
- Self-Check Quiz

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**MINI Lab**

Three squares with sides 3, 4, and 5 units are used to form the right triangle shown.

1. Find the area of each square.
2. How are the squares of the sides related to the areas of the squares?
3. Find the sum of the areas of the two smaller squares. How does the sum compare to the area of the larger square?
4. Use grid paper to cut out three squares with sides 5, 12, and 13 units. Form a right triangle with these squares. Compare the sum of the areas of the two smaller squares with the area of the larger square.

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In a right triangle, the sides have special names.

- The two sides that form the right angle are the **legs**.
- The side opposite the right angle is the **hypotenuse**. It is the longest side of the triangle.

The **Pythagorean Theorem** describes the relationship between the length of the hypotenuse and the lengths of the legs.

**Pythagorean Theorem**

<table>
<thead>
<tr>
<th>Words</th>
<th>Model</th>
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<tbody>
<tr>
<td>In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.</td>
<td>[c^2 = a^2 + b^2]</td>
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When using the Pythagorean Theorem, you will encounter equations that involve square roots. Every positive number has both a positive and a negative square root. By the definition of square roots, if \(n^2 = a\), then \(n = \pm \sqrt{a}\). The notation \(\pm \sqrt{a}\) indicates both the positive and negative square root of a number. You can use this relationship to solve equations that involve squares.
**EXAMPLE**

Find the length of the hypotenuse of the triangle.

\[ c^2 = a^2 + b^2 \]  
**Pythagorean Theorem**

\[ c^2 = 8^2 + 4^2 \]  
Replace \( a \) with 8 and \( b \) with 4.

\[ c^2 = 64 + 16 \]  
Evaluate \( 8^2 \) and \( 4^2 \).

\[ c^2 = 80 \]  
Add.

\[ c = \pm \sqrt{80} \]  
Definition of square root

\[ c \approx \pm 8.9 \]  
Simplify.

The length of the hypotenuse is about 8.9 feet.

**CHECK Your Progress**

a. Find the length of the hypotenuse of a right triangle with legs 5 yards and 7 yards. Round to the nearest tenth.

**Real-World EXAMPLE**

**SCUBA DIVING** A scuba diver dove 14 feet below the surface. Then, he swam 16 feet toward a coral formation. How far is the diver from his boat?

The diver’s distance from the boat is the hypotenuse of a right triangle. Write and solve an equation for \( x \).

\[ c^2 = a^2 + b^2 \]  
**Pythagorean Theorem**

\[ x^2 = 14^2 + 16^2 \]  
Replace \( c \) with \( x \), \( a \) with 14, and \( b \) with 16.

\[ x^2 = 196 + 256 \]  
Evaluate \( 14^2 \) and \( 16^2 \).

\[ x^2 = 452 \]  
Add.

\[ x = \pm \sqrt{452} \]  
Definition of square root

\[ x \approx \pm 21.3 \]  
Simplify.

The diver’s distance from the boat is about 21.3 feet.

**CHECK Your Progress**

b. **SOFTBALL** A softball diamond is a square measuring 60 feet on each side. How far does a player on second base throw when she throws from second base to home? Round to the nearest tenth.
Mr. Thomson created a mosaic tile in the shape of a square to place in his kitchen.

Which is closest to the length of the diagonal of the tile?

A 10 in.  C 15 in.
B 13 in.  D 17 in.

Read the Item
You need to use the Pythagorean Theorem to find the length of the diagonal.
Solve the Item

\[ c^2 = a^2 + b^2 \]

Pythagorean Theorem

\[ c^2 = 9^2 + 9^2 \]

Replace \( a \) with 9 and \( b \) with 9.

\[ c^2 = 81 + 81 \]

Evaluate \( 9^2 \) and \( 9^2 \).

\[ c^2 = 162 \]

Add.

\[ c = \pm \sqrt{162} \]

Definition of square root

\[ c \approx \pm 12.7 \]

Simplify.

The length is about 12.7 inches.

The answer choice closest to 12.7 inches is 13 inches. So, the answer is B.

f. A painter leans a ladder against the side of a building. How far from the bottom of the building is the top of the ladder?

F 38.2 ft  H 21.8 ft

G 28.0 ft  J 20.0 ft

***Example 2*** (p. 641)

5. **ARCHITECTURE** What is the width of the fence gate shown at the right? Round to the nearest tenth.

***Example 4*** (pp. 642–643)

6. **MULTIPLE CHOICE** A company designed a public play area in the shape of a square. The play area will include a pathway, as shown. Which is closest to the length of the pathway?

A 100 yd  
B 125 yd  
C 140 yd  
D 175 yd

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**Check Your Understanding**

Examples 1, 3 (pp. 641–642)

Find the missing measure of each triangle. Round to the nearest tenth if necessary.

1. \[ \triangle \]

\[ 24 \text{ mm} \]

\[ 10 \text{ mm} \]

\[ c \text{ mm} \]

2. \[ \triangle \]

\[ 31 \text{ in.} \]

\[ 19 \text{ in.} \]

\[ a \text{ in.} \]

3. \( b = 21 \text{ cm}, c = 28 \text{ cm} \)

4. \( a = 11 \text{ yd}, b = 12 \text{ yd} \)
Find the missing measure of each triangle. Round to the nearest tenth if necessary.

7. \( \triangle \) with sides 8 m, 14 m, and unknown side \( c \) m.

8. \( \triangle \) with sides 21 in. and 28 in., and unknown side \( c \) in.

9. \( \triangle \) with sides 5 m and unknown side \( a \) m.

10. \( \triangle \) with sides 14 cm and unknown side \( b \) cm.

11. \( \triangle \) with sides 4.6 ft and 2.8 ft, and unknown side \( c \) ft.

12. \( \triangle \) with sides 8.9 mm and unknown side \( c \) mm.

13. \( a = 2.4 \) yd, \( c = 3.7 \) yd

14. \( b = 8.5 \) m, \( c = 10.4 \) m

15. \( a = 7 \) in., \( b = 24 \) in.

16. \( a = 13.5 \) mm, \( b = 18 \) mm

MEASUREMENT For Exercises 17 and 18, find each distance to the nearest tenth.

17. School to bank: 3.2 mi, 4.6 mi. Find \( x \) mi.

18. Tree to bench: 14.5 ft, 12.8 ft. Find \( x \) ft.

SPORTS For Exercises 19 and 20, find the length or width of each piece of sports equipment. Round to the nearest tenth.


20. Table tennis net: 36 in. width, 54.6 in. length. Find \( x \) in.

21. MEASUREMENT A barn door is 10 feet wide and 15 feet tall. A square plank 16 feet on each side must be taken through the doorway. Can the plank fit through the doorway? Justify your answer.

22. MEASUREMENT On a weekend trip around California, Sydney left her home in Modesto and drove 75 miles east to Yosemite National Park, then 70 miles south to Fresno, and finally 110 miles west to Monterey Bay. About how far is she from her starting point? Justify your answer with a drawing.
23. **CHALLENGE** What is the length of the diagonal shown in the cube at the right?

![Cube Diagram]

24. **FIND THE ERROR** Marcus and Aisha are writing an equation to find the missing measure of the triangle at the right. Who is correct? Explain.

Marcus: \(x^2 = 21^2 + 8^2\)

Aisha: \(21^2 = b^2 + x^2\)

![Marcus and Aisha Images]

25. **WRITING IN MATH** Write a problem about a real-world situation in which you would use the Pythagorean Theorem.

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**TEST PRACTICE**

26. Which triangle has sides \(a\), \(b\), and \(c\) so that the relationship \(a^2 + b^2 = c^2\) is true?

![Triangle Images]

27. An isosceles right triangle has legs that are each 8 inches long. About how long is the hypotenuse?

- F 12.8 inches
- G 11.3 inches
- H 8 inches
- J 4 inches

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**Spiral Review**

28. **ESTIMATION** Which is closer to \(\sqrt{55}\) : 7 or 8? (Lesson 12-1)

29. **MEASUREMENT** A cylinder-shaped popcorn tin has a height of 1.5 feet and a diameter of 10 inches. Find the volume to the nearest cubic inch. (Lesson 11-10)

Write each percent as a decimal. (Lesson 4-7)

30. 45%  
31. 8%  
32. 124%  
33. 265%

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**GET READY for the Next Lesson**

34. **PREREQUISITE SKILL** The average person takes about 15 breaths per minute. At this rate, how many breaths does the average person take in one week? Use the solve a simpler problem strategy. (Lesson 11-5)